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I.Sc. Class-XI, Date- 21-04-2020
Permutations and Combinations.
Miscellaneous Exercise

Q.1. How many words, with or without meaning can be formed each of 2 vowels and 3 consonants from the letters of the word 'DAUGHTER'?

So. We have formed from the letters of the word Daughter.
Number of vowel = A, E, U = 3
Number of consonants = D, G, H, T, R = 5
2 vowels out of 3 vowels can be selected = $^3C_2$ ways
3 consonants out of 5 consonants can be selected = $^5C_3$ ways
And 2 vowels and 3 consonants can be ~~selected~~ arranged in $\underline{|5}$ ways
Hence, by Fundamental principle of Counting, total number of ways = $^3C_2 \times {^5C_3} \times \underline{|5} = \frac{\underline{|3}}{\underline{|2}\,\underline{|3-2}} \times \frac{\underline{|5}}{\underline{|3}\,\underline{|5-3}} \times \underline{|5}$

$= \frac{\underline{|3}}{\underline{|2}\,\underline{|1}} \times \frac{\underline{|5}}{\underline{|3}\,\underline{|2}} \times \underline{|5} = \frac{3\,\underline{|2}}{\underline{|2}} \times \frac{5 \times 4 \times \underline{|3}}{\underline{|3} \times 2 \times 1} \times 5 \times 4 \times 3 \times 2 \times 1$

$= 3 \times 10 \times 120 = 3600$ ways $[\underline{|1} = 1,\ \underline{|5} = 5 \times 4 \times 3 \times 2 \times 1 = 120]$

Q.2. How many words, with or without meaning, can be formed using all the letters of the word 'EQUATION' at a time so that the vowels and consonants occur together?

Soln. Given word is EQUATION.

Number of vowels = E, U, A, I, O = 5
Number of consonants = T, Q, N = 3
These vowels and consonants can be arranged in $\underline{|2}$ ways.
Hence, by FPC, total number of ways
$= \underline{|5} \times \underline{|3} \times \underline{|2} = 5 \times 4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1 \times 2 \times 1 = 1440$ Ans

B.Sc. Class-XI Permutation and Combination
Date - 21-04-2020

Miscellaneous Exercise

Q.3. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of
(I) exactly 3 girls? (II) atleast 3 girls? ~~(iii) atmost 3 girls~~

Soln: (I) Out of 7 persons, we want to select exactly 3 girls i.e. remaining 4 persons will be boys.
4 boys out of 9 boys can be selected in $^9C_4$ ways.
3 girls out of 4 girls can be selected in $^4C_3$ ways.
Hence, by Fundamental Principle of Counting, total number of ways for making the committee

$$= {}^9C_4 \times {}^4C_3 = \frac{\lfloor 9}{\lfloor 4 \lfloor 9-4} \times \frac{\lfloor 4}{\lfloor 3 \lfloor 4-3} \quad \left[\because {}^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}\right]$$

$$= \frac{9\times 8\times 7\times 6\times \lfloor 5}{4\times 3\times 2\times 1\times \lfloor 5} \times \frac{4 \lfloor 3}{\lfloor 3 \times \lfloor 1} = 9\times 7\times 2\times \frac{4}{1} = 9\times 7\times 8 = 504 \text{ ways.}$$

(II) We have to select atleast 3 girls.
In first case, 4 boys out of 9 boys can be selected in $^9C_4$ ways and 3 girls out of 4 girls can be selected in $^4C_3$ ways.
Hence, by Fundamental Principle of Counting, total number of ways in this case $= {}^9C_4 \times {}^4C_3 = \frac{\lfloor 9}{\lfloor 4 \lfloor 9-4} \times \frac{\lfloor 4}{\lfloor 3 \lfloor 4-3}$

$$= \frac{\lfloor 9}{\lfloor 4 \lfloor 5} \times \frac{\lfloor 4}{\lfloor 3 \lfloor 1}$$

$$= \frac{9\times 8\times 7\times 6\times \lfloor 5}{4\times 3\times 2\times 1\times \lfloor 5} \times \frac{4\times \lfloor 3}{\lfloor 3}$$

$$= 9\times 2\times 7\times 4 = 504 \text{ ways.}$$

In second case, 3 boys out of 9 boys can be select in $^9C_3$ ways
And, 4 girls out of 4 girls can be select in $^4C_4$ ways.
Hence, by Fundamental Principle of Counting, total number of ways in this case $= {}^9C_3 \times {}^4C_4 = \frac{9\times 8\times 7\times \lfloor 6}{\lfloor 3\times \lfloor 9-3} \times \frac{\lfloor 4}{\lfloor 4 \lfloor 4-4}$

$$= \frac{9\times 8\times 7\times \lfloor 6}{3\times 2\times 1\times \lfloor 6} \times \frac{1}{\lfloor 0} = 84.$$

Hence, by fundamental Principle of Counting, total number of ways
$= 504 + 84 = 588$ Ans.